12.19 Area Between a Curve and Its Tangent Line

ASSIGNMENT: In this project,[4] you will find the tangent line to the graph of a function for which the area between the curve and the tangent line is a minimum.

1. Pick a function $y = f(x)$ which is everywhere concave up or everywhere concave down, such as $y = f(x) = -x^2$. NOTE: If the concavity changes, then the tangent line might cross the curve, which we don't want.

2. Find its tangent line at a general point $x = p$.

3. Compute the area between the curve and its tangent line at $x = p$ above the interval $0 \le x \le 1$. Label it `Area`.

4. Find the point $x = p_{\min}$ for which `Area` is a minimum. Be sure to apply the Second Derivative Test to verify that your critical point is a minimum.

5. Plot the curve and the tangent line for several values of p in $[0, 1]$ including the minimum. Plot the `Area` function.

6. Repeat steps 1-5 for three or more other functions $f(x)$. Use interesting functions, not just polynomials, and check the concavity on the interval $[0, 1]$. Be sure to try functions which are concave up as well as concave down.

7. What do you conjecture?

8. Prove your conjecture by repeating steps 1-4 for an undefined function `f:=g(x)`, once assuming g is concave up and once assuming g is concave down. Before solving for p you will need to give names to the derivatives of g using, for example, `subs(diff(g(p),p,p)=ddg, ...)`.

9. What happens to your conjecture and proof if you change the interval from $[0, 1]$ to $[a, b]$?

[4] The idea for this project was originally suggested by Carol Scheftic, Cal. Poly. St. Univ.